13.2 Videos Guide

13.2a

- Derivative of a vector function
  - $\frac{d\mathbf{r}}{dt} = \mathbf{r}'(t) = \langle f'(t), g'(t), h'(t) \rangle = f'(t)\mathbf{i} + g'(t)\mathbf{j} + h'(t)\mathbf{k}$
- Product rules with vector functions
- The unit tangent vector: $\mathbf{T}(t) = \frac{\mathbf{r}'(t)}{|\mathbf{r}'(t)|}$

Exercises:

13.2b

- Sketch the plane curve with the given vector equation.
  (b) Find $\mathbf{r}'(t)$.
  (c) Sketch the position vector $\mathbf{r}(t)$ and the tangent vector $\mathbf{r}'(t)$ for the given value of t.
  $\mathbf{r}(t) = e^t\mathbf{i} + 2t\mathbf{j}, \qquad t = 0$
- Find the derivative of the vector function $\mathbf{r}(t) = \sin^2 at\,\mathbf{i} + te^{bt}\,\mathbf{j} + \cos^2 ct\,\mathbf{k}$.

13.2c

- Find the unit tangent vector $\mathbf{T}(t)$ at the point with the given value of the parameter t.
  $\mathbf{r}(t) = \langle \tan^{-1} t\,, 2e^{2t}, 8te^t \rangle, \qquad t = 0$
- Find parametric equations for the tangent line to the curve with the given parametric equations at the specified point.
  $x = \ln(t+1),\ y = t\cos 2t,\ z = 2^t, \qquad (0, 0, 1)$

13.2d

- Proof that if $|\mathbf{r}(t)| = c$, a constant, then $\mathbf{r}'(t)$ is orthogonal to $\mathbf{r}(t)$ for all t

13.2e

- Integrals of vector functions
  - Indefinite: $\int \mathbf{r}(t)\,dt = \int f(t)\mathbf{i} + \int g(t)\mathbf{j} + \int h(t)\mathbf{k} + \mathbf{C}$
  - Definite: $\int_a^b \mathbf{r}(t)\,dt = \int_a^b f(t)\mathbf{i}\,dt + \int_a^b g(t)\mathbf{j}\,dt + \int_a^b h(t)\mathbf{k}\,dt$

Exercise:

- Evaluate the integral.
  $\int_1^4 \left(2t^{3/2}\,\mathbf{i} + (t+1)\sqrt{t}\,\mathbf{k}\right) dt$